

On the Charge-Dependent Matrix Element Between the 9.516 MeV $4^+ T = 1$ Level and the 8.439 MeV $4^+ T = 0$ Level of ^{24}Mg

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A recent measurement of the β - γ circular polarization asymmetry parameter A on the isospin-forbidden β^+ decay of the ground state of ^{24}Al to the 8.438 MeV $4^+ T = 0$ state of ^{24}Mg yields a charge-dependent matrix element $\langle V_{\text{CD}} \rangle_{\text{Mg}}$ between the 9.516 MeV $4^+ T = 1$ state and the 8.439 MeV $4^+ T = 0$ state of ^{24}Mg of (106 ± 40) keV, which is the largest measured value obtained so far in β decay. Our calculation using Nilsson's wavefunctions and a spherical Coulomb potential results in a value of 87 keV for $\langle V_{\text{CD}} \rangle_{\text{Mg}}$. Variation of the value of $\langle V_{\text{CD}} \rangle_{\text{Mg}}$ against the deformation parameter β shows that theory agree very well with the measured $\langle V_{\text{CD}} \rangle_{\text{Mg}}$ for β around 0.5.

Introduction

Recently a measurement [1] of the β - γ circular polarization asymmetry parameter A (defined by $W(\theta, \tau) = 1 + \tau(v/c) A \cos \theta$, where $\tau = +1$ or -1 corresponding to right or left circular polarization, respectively) was made on the isospin-forbidden β^+ decay of the ground state of ^{24}Al to the 8.439 MeV $4^+ T = 0$ state of ^{24}Mg . The measured value of $A = -0.145 \pm 0.030$ yields a charge-dependent matrix element between the 9.516 MeV $4^+ T = 1$ and 8.439 MeV $4^+ T = 0$ levels of ^{24}Mg , namely

$$\begin{aligned} \langle (8.439) 4^+ T = 0 T_z = 0 | V_{\text{CD}} | (9.516) 4^+ T = 1 T_z = 0 \rangle \\ \equiv \langle V_{\text{CD}} \rangle_{\text{Mg}} = (106 \pm 40) \text{ keV} \quad (1) \end{aligned}$$

which is the largest value obtained so far for a β decay.

Figure 1 shows the frequency distribution of charge-dependent matrix elements $\langle V_{\text{CD}} \rangle$ from a recent compilation [2]. Most of the $\langle V_{\text{CD}} \rangle$ values are below 10 keV with a small number between 10–20 keV. The largest two values (56 ± 12 keV for ^{57}Ni and 41.7 ± 1.1 keV for ^{64}Ga) were omitted in the figure. It is therefore interesting to do a calculation for $\langle V_{\text{CD}} \rangle_{\text{Mg}}$ of ^{24}Mg , using only the Coulomb force for the charge-dependent interaction. Previous calculations [3] on various isospin-forbidden β decays

using the Nilsson model [4] and the Coulomb interaction gave reasonable agreement with experiment.

Calculation and Results

It is well-known that ^{24}Mg is rather deformed and the Nilsson model has been used in the classification of the levels of ^{24}Mg into rotational bands [5]. For deformations [5] around $\beta = 0.4$, the ordering of the Nilsson orbitals in increasing energy after the ^{16}O core is: # 6 $\frac{1}{2}^+$ [220], # 7 $\frac{3}{2}^+$ [211] and # 5 $\frac{5}{2}^+$ [202]. The wavefunction $| (9.516) 4^+ T = 1, T_z = 0 \rangle$ of the 9.516 MeV level of ^{24}Mg is the analogue of the ground state $| 4^+ T = 1, T_z = 1 \rangle$ of ^{24}Al . Therefore $| (9.516) 4^+ T = 1, T_z = 0 \rangle = T_- | 4^+ T = 1, T_z = 1 \rangle$ and

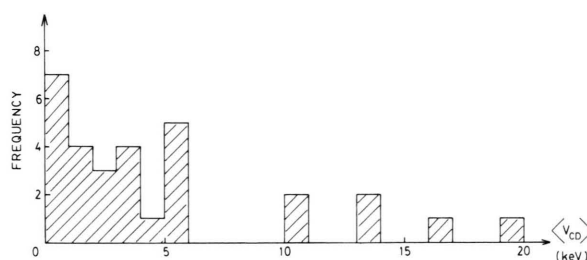


Fig. 1. Frequency distribution of the experimental charge-dependent matrix element $\langle V_{\text{CD}} \rangle$ from [2]. The two large values (^{57}Ni and ^{64}Ga) were omitted in the plot.

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is given by

$$(9.516) 4^+, T = 1, T_z = 0 \rangle \quad (2)$$

$$= \frac{1}{\sqrt{2}} \left(\begin{array}{c} \frac{5}{2}^+ [202] \text{ --- } \times \\ \frac{3}{2}^+ [211] \text{ --- } \circ \end{array} \begin{array}{|c|c|} \hline p & n \\ \hline \end{array} + \begin{array}{c} \text{--- } \circ \\ \text{--- } \times \end{array} \begin{array}{|c|c|} \hline p & n \\ \hline \end{array} \right)$$

The wavefunction of the 8.439 MeV level of ^{24}Mg , the antianalogue state, is given by

$$(8.439) 4^+, T = 0, T_z = 0 \rangle \quad (3)$$

$$= \frac{1}{\sqrt{2}} \left(\begin{array}{c} \frac{5}{2}^+ [202] \text{ --- } \circ \\ \frac{3}{2}^+ [211] \text{ --- } \times \end{array} \begin{array}{|c|c|} \hline p & n \\ \hline \end{array} - \begin{array}{c} \text{--- } \times \\ \text{--- } \circ \end{array} \begin{array}{|c|c|} \hline p & n \\ \hline \end{array} \right)$$

In this calculation, the Coulomb potential is taken as a one-body spheroidal potential which may be written as [6]

$$V_{\text{CD}} = \frac{(Z-1)e^2}{R} \left[\frac{3}{2} - \frac{1}{2} (r/R)^2 \right] + a(r/R)^2 Y_{20} \quad \text{for } r < R, \quad (4)$$

$$V_{\text{CD}} = \frac{(Z-1)e^2}{r} + a(R/r)^3 Y_{20} \quad \text{for } r > R,$$

where R is the radius of the nucleus and a is related to the Bohr deformation parameter β by

$$a = \frac{3}{5} \beta \frac{(Z-1)e^2}{R}. \quad (5)$$

using $\beta = 0.4$ and Nilsson's wavefunctions, we obtained

$$\begin{aligned} \langle V_{\text{CD}} \rangle_{\text{Mg}} &\equiv \langle (8.439) 4^+ T = 0, T_z = 0 | \\ &\quad \cdot V_{\text{CD}} | (9.516) 4^+ T = 1, T_z = 0 \rangle \\ &= \frac{1}{2} \{ \langle \frac{5}{2}^+ [202] | V_{\text{CD}} | \frac{5}{2}^+ [202] \rangle \\ &\quad - \langle \frac{3}{2}^+ [211] | V_{\text{CD}} | \frac{3}{2}^+ [211] \rangle \} \\ &= 87 \text{ keV}, \end{aligned} \quad (6)$$

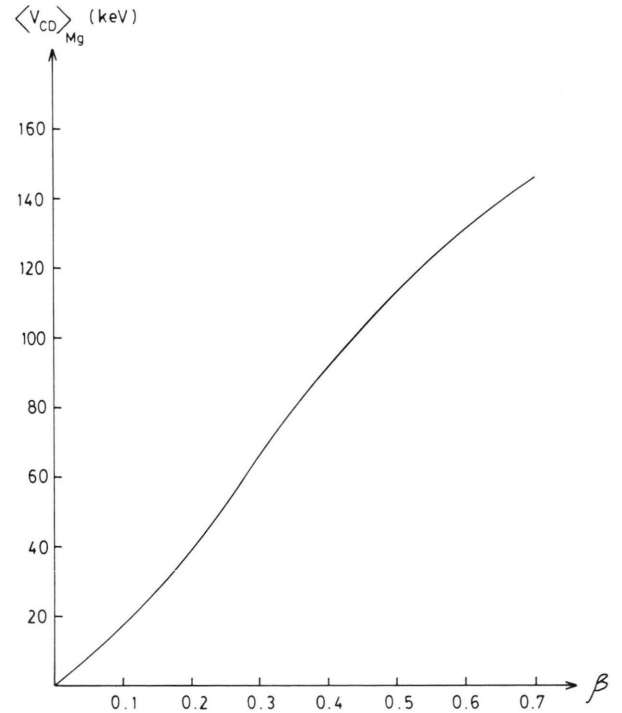


Fig. 2. Variation of the theoretical charge-dependent matrix element $\langle V_{\text{CD}} \rangle_{\text{Mg}}$ of ^{24}Mg as a function of the deformation parameter β .

which is in reasonable agreement with the experimental value.

The variation of the value of the charge-dependent matrix element $\langle V_{\text{CD}} \rangle_{\text{Mg}}$ against the deformation parameter β should be investigated. The results of such a calculation are presented in Figure 2. It shows an increasing value of $\langle V_{\text{CD}} \rangle_{\text{Mg}}$ as β increases, and therefore the agreement with the experimental value of $(106 \pm 40) \text{ keV}$ is even better for β around 0.5. This seems to indicate that the Coulomb interaction alone might be sufficient to explain isospin mixing. Furthermore, as with the case [7] of ^{64}Ga , the exceptionally large value of the charge-dependent matrix element is due to the similarity of space and spin wavefunctions of the analogue and antianalogue states.

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