On the Charge-Dependent Matrix Element Between the 9.516 MeV 4^+ $T = 1$ Level and the 8.439 MeV 4^+ $T = 0$ Level of ^{24}Mg

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A recent measurement of the $\beta - \gamma$ circular polarization asymmetry parameter A on the isospin-forbidden β^+ decay of the ground state of ^{24}Al to the 8.438 MeV 4^+ $T = 0$ state of ^{24}Mg yields a charge-dependent matrix element $\langle V_{CD} \rangle_{Mg}$ between the 9.516 MeV $4^+ T = 1$ state and the 8.439 MeV $4^+ T = 0$ state of ^{24}Mg of (106 ± 40) keV, which is the largest measured value obtained so far in β decay. Our calculation using Nilsson's wavefunctions and a spherical Coulomb potential results in a value of 87 keV for $\langle V_{CD} \rangle_{Mg}$. Variation of the value of $\langle V_{CD} \rangle_{Mg}$ against the deformation parameter β shows that theory agree very well with the measured $\langle V_{CD} \rangle_{Mg}$ for β around 0.5.

Introduction

Recently a measurement [1] of the $\beta - \gamma$ circular polarization asymmetry parameter A (defined by $W(\theta, \tau) = 1 + \tau(v/c) A \cos\theta$, where $\tau = +1$ or -1 corresponding to right of left circular polarization, respectively) was made on the isospin-forbidden β^+ decay of the ground state of ^{24}Al to the 8.439 MeV $4^+ T = 0$ state of ^{24}Mg. The measured value of $A = -0.145 \pm 0.030$ yields a charge-dependent matrix element between the 9.516 MeV $4^+ T = 1$ and 8.439 MeV $4^+ T = 0$ levels of ^{24}Mg, namely

$$\langle (8.439)\, 4^+ T = 0\ T_z = 0 | V_{CD} | (9.516)\, 4^+ T = 1\ T_z = 0 \rangle \equiv \langle V_{CD} \rangle_{Mg} = (106 \pm 40)\ \text{keV} \quad (1)$$

which is the largest value obtained so far for a β decay.

Figure 1 shows the frequency distribution of charge-dependent matrix elements $\langle V_{CD} \rangle$ from a recent compilation [2]. Most of the $\langle V_{CD} \rangle$ values are below 10 keV with a small number between 10–20 keV. The largest two values (56 ± 12 keV for ^{57}Ni and 41.7 ± 1.1 keV for ^{64}Ga) were omitted in the figure. It is therefore interesting to do a calculation for $\langle V_{CD} \rangle_{Mg}$ of ^{24}Mg, using only the Coulomb force for the charge-dependent interaction. Previous calculations [3] on various isospin-forbidden β decays using the Nilsson model [4] and the Coulomb interaction gave reasonable agreement with experiment.

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Calculation and Results

It is well-known that ^{24}Mg is rather deformed and the Nilsson model has been used in the classification of the levels of ^{24}Mg into rotational bands [5]. For deformations [5] around $\beta = 0.4$, the ordering of the Nilsson orbitals in increasing energy after the ^{16}O core is: # 6 $\frac{1}{2}^+$ [220], # 7 $\frac{3}{2}^+$ [211] and # 5 $\frac{5}{2}^+$ [202]. The wavefunction $|(9.516)\, 4^+ T = 1, T_z = 0\rangle$ of the 9.516 MeV level of ^{24}Mg is the analogue of the ground state $|4^+ T = 1, T_z = 1\rangle$ of ^{24}Al. Therefore $|(9.516)\, 4^+ T = 1, T_z = 0\rangle = T_- |4^+ T = 1, T_z = 1\rangle$ and

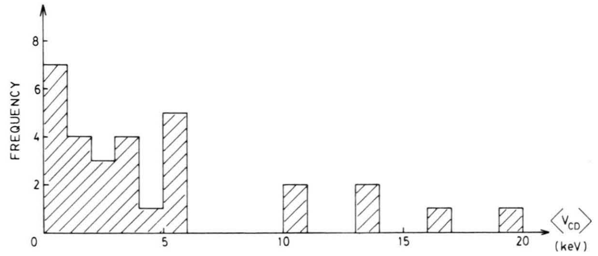

Fig. 1. Frequency distribution of the experimental charge-dependent matrix element $\langle V_{CD} \rangle$ from [2]. The two large values (^{57}Ni and ^{64}Ga) were omitted in the plot.

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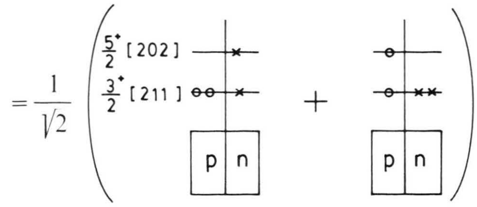
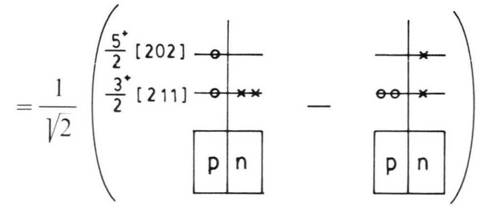
is given by

$|(9.516)\ 4^+,\ T=1,\ T_z=0\rangle$ (2)



The wavefunction of the 8.439 MeV level of ^{24}Mg, the antianalogue state, is given by

$|(8.439)\ 4^+,\ T=0,\ T_z=0\rangle$ (3)



In this calculation, the Coulomb potential is taken as a one-body spheriodal potential which may be written as [6]

$$V_{CD} = \frac{(Z-1)\,e^2}{R}\left[\tfrac{3}{2} - \tfrac{1}{2}(r/R)^2\right] + a(r/R)^2\,Y_{20} \quad \text{for} \quad r < R,$$

$$V_{CD} = \frac{(Z-1)\,e^2}{r} + a(R/r)^3\,Y_{20} \quad \text{for} \quad r > R, \tag{4}$$

where R is the radius of the nucleus and a is related to the Bohr deformation parameter β by

$$a = \frac{3}{5}\,\beta\,\frac{(Z-1)\,e^2}{R}. \tag{5}$$

using $\beta = 0.4$ and Nilsson's wavefunctions, we obtained

$$\begin{aligned}\langle V_{CD}\rangle_{Mg} &\equiv \langle (8.439)\ 4^+ T=0,\ T_z=0 | \\ &\quad \cdot V_{CD}\,|(9.516)\ 4^+ T=1,\ T_z=0\rangle \\ &= \tfrac{1}{2}\{\langle \tfrac{5}{2}^+[202]|\,V_{CD}\,|\tfrac{5}{2}^+[202]\rangle \\ &\qquad - \langle \tfrac{3}{2}^+[211]|\,V_{CD}\,|\tfrac{3}{2}^+[211]\rangle\} \\ &= 87\ \text{keV},\end{aligned} \tag{6}$$

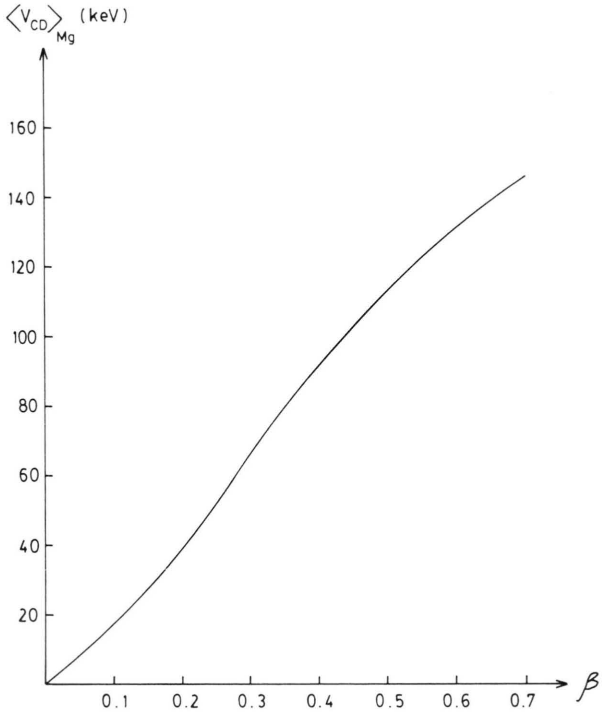


Fig. 2. Variation of the theoretical charge-dependent matrix element $\langle V_{CD}\rangle_{Mg}$ of ^{24}Mg as a function of the deformation parameter β.

which is in reasonable agreement with the experimental value.

The variation of the value of the charge-dependent matrix element $\langle V_{CD}\rangle_{Mg}$ against the deformation parameter β should be investigated. The results of such a calculation are presented in Figure 2. It shows an increasing value of $\langle V_{CD}\rangle_{Mg}$ as β increases, and therefore the agreement with the experimental value of (106 ± 40) keV is even better for β around 0.5. This seems to indicate that the Coulomb interaction alone might be sufficient to explain isospin mixing. Furthermore, as with the case [7] of ^{64}Ga, the exceptionally large value of the charge-dependent matrix element is due to the similarity of space and spin wavefunctions of the analogue and antianalogue states.

Acknowledgement

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